Sample Unit Years 8–10

Graphical approach to modelling with quadratics

MATHEMATICS Levels 5 and 6

Contents

Victoria The Place To Be

Published by

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

41 St Andrews Place East Melbourne Victoria 3002 Australia

TELEPHONE +61 3 9651 4300 FACSIMILE +61 3 9651 4324

EMAIL vcaa@edumail.vic.gov.au

WEB www.vcaa.vic.edu.au

Tr

Teacher Resource

Introduction

In *Graphical approach to modelling with quadratics* students undertake an exploration of parabolic shapes in various real life situations and their mathematical modelling.

The unit provides students with skills in:

- recognising parabolic curves and their features
- plotting quadratic functions and identifying key features
- applying transformations to basic parabolas
- identifying the relationship between the shape of a quadratic function and the turning point form of its equation.

These sample activities are best used as a sequential unit of work. Activities include:

- investigating parabolic curves in design and construction
- exploring the basic parabolic shape
- investigating the factors that affect the shape of a graph
- applying mathematical modelling principles to practical contexts.

Before commencing this unit, refer to the **Teacher notes** section (pages 7–18). This section contains advice about:

- using the student worksheets
- preparation
- teaching approaches
- assessing this unit.

The **Teacher notes** section also includes a **Unit summary** (page 18) of relevant CSF learning outcomes, key competencies and ICT skills. Relevant learning outcomes and key competencies are also detailed in the notes for each activity.

Unit objectives

Students are provided with opportunities to:

- appreciate that parabolic curves arise in a broad range of situations
- describe the key properties of a parabolic curve
- identify a quadratic equation
- understand the effect of a, h and k on the graph of $y = a(x - h)^2 + k$
- determine the rule of a quadratic function from its graph
- apply their knowledge to develop models using graphs of quadratic functions in real life situations.

Curriculum focus

The CSF level 5 and 6 Mathematics Curriculum focus statements describe the scope of teaching and learning in Mathematics from Years 8–10. These are available online at: www.vcaa.vic.edu.au.

At these levels students work with increasingly general representations of material in each strand. Students develop increasing awareness of how mathematics is used to solve a broad range of problems and use a variety of modelling and problem-solving approaches to enable investigation of objects and activities from a mathematical perspective. The activities in *Graphical approach to modelling with quadratics* encourage students to investigate the use of mathematics in modelling real events and situations.

Activities

The unit aims to raise students' awareness of the wide spread applications of parabolic curves and the shape and symmetry of the graph of the basic quadratic functions, $y = x^2$ and $y = {}^-x^2$.

Students firstly develop familiarisation with parabolas using developmental activities, including Internet research, before tackling applications.

Students investigate the effects of a, h and k on the graph of $y = a(x – h)^2 + k$ with the support of ICT and/or graphics calculator technology and determine the matching quadratic equations to given graphs. The final activities draw together the work of the unit by engaging students in using parabolic curves to model practical situations.

Activity 1: Exploring parabolic curves and the basic shape of the graph of quadratic functions

Worksheet 1.1 (pages 22–23), with its Internet research, aims to draw students' attention to the fact that parabolic curves have been used for hundreds of years in architecture and design and are currently still being used in the design of bridges, buildings, satellite dishes, headlights, solar cookers and other appliances. The search helps students put this area of mathematics into a 'real life' context and appreciate some of its practical applications.

The second Internet search (quadratics and parabolas) highlights the fact that quadratic functions enable us to define a parabolic curve and that the study of quadratic functions involves algebraic calculations. It is not intended that students investigate the quadratic functions in any detail while doing the search. Various search options are included to stimulate and show students the numerous areas in which parabolic curves are found.

Worksheet 1.2 (pages 24–26) requires students to graph the basic shapes of $y = x^2$ and $y = {}^-x^2$, taking note of features such as the number of turning points, symmetry and the increasing or decreasing nature of $y = x^2$ and $y = {}^-x^2$ respectively.

In Worksheet 1.3 (page 27) students reflect on the nature of the functions they are presented with and the corresponding equations and curves that are generated.

Tr

Teacher Resource

Assessing Activity 1

Students can present the findings of the investigation of parabolic curves in a PowerPoint or poster format. Solutions to Worksheets 1.2 and 1.3 are in **Assessing this unit** (page 9).

CSF learning outcomes

Mental computation and estimation

5.1 Extend the use of basic number facts to mentally compute operations on fractions and decimals, and squares and square roots. MANUM501
5.3 Use estimation strategies to check computations with fractions and decimals. MANUM503

Computation and applying number

5.3 Carry out the four operations in cases where both positive and negative integers are involved. MANUC503

Expressing generality

5.1 Develop, interpret and simplify mathematical expressions which describe rules for relationships and mensuration formulas. MAALE501
6.3 Construct and interpret rules for linear, quadratic, reciprocal and exponential relationships. MAALE603

Function

5.1 Use ordered pairs to locate and describe the positions of points on a Cartesian coordinate grid. MAALF501
5.3 Plot graphs of linear and other simple functions and use linear functions to model data. MAALF503
6.1 Plot and sketch graphs of linear, quadratic and exponential functions and other simple functions. MAALF601
6.2 Interpret graphs of linear, quadratic, exponential functions and other simple functions. MAALF602

Mathematical reasoning

5.1 Make, test and modify conjectures. MARSR501
5.3 Use and interpret simple mathematical models and make judgments about the accuracy and suitability of the results obtained by using mathematical models. MARSR503

Strategies for investigation

5.1 Generate mathematical questions for inquiry from presented data, familiar contexts and the experience gained from inquiries in relation to previous tasks and problems. MARSS501

Key competencies

1 Collecting, analysing and organising information
2 Communicating ideas and information
3 Planning and organising activities
4 Working with others and in teams
5 Using mathematical ideas and techniques

Tr

Teacher Resource

6 Solving problems
7 Using technology

Activity 2: Investigating the effect of *a*, *h* and *k* on the graph of $y = a(x - h)^2 + k$

Using the four worksheets in this activity (Worksheets 2.1, 2.2, 2.3 and 2.4, see pages 28–43) students investigate the effect of *a*, *h* and *k* on the graph of a quadratic function in completed square form. Worksheets 2.1 and 2.3 require students to complete tables of values and plot graphs by hand.

Worksheets 2.2 and 2.4 investigate the rule of the quadratic function for a given parabola by locating the coordinates of the vertex to determine *h* and *k*. An additional point (*x*, *y*) is used to determine *a*.

Assessing Activity 2

Students record answers to worksheet questions. Worksheet solutions are in **Assessing this unit** (page 9).
The following questions are particularly useful in indicating student understanding and progress and can be formally assessed.

- Worksheet 2.1 question 4 parts b) and c)
- Worksheet 2.1 questions 5 and 6
- Worksheet 2.2 question 1
- Worksheet 2.3 questions 4 and 6
- Worksheet 2.4 question 1.

CSF learning outcomes

Mental computation and estimation

5.1 Extend the use of basic number facts to mentally compute operations on fractions and decimals, and squares and square roots. MANUM501

Computation and applying number

5.3 Carry out the four operations in cases where both positive and negative integers are involved. MANUC503
6.4 Choose and use appropriate technology to assist with the computations necessary to solve numerical problems at this level. MANUC604

Expressing generality

5.1 Develop, interpret and simplify mathematical expressions which describe rules for relationships and mensuration formulas. MAALE501
6.1 Use methods of algebraic manipulation to rearrange and simplify mathematical expressions and change the subject of a formula. MAALE601
6.3 Construct and interpret rules for linear, quadratic, reciprocal and exponential relationships. MAALE603

Tr
Teacher Resource

Function

6.1 Plot and sketch graphs of linear, quadratic and exponential functions and other simple functions. MAALF601
6.2 Interpret graphs of linear, quadratic, exponential functions and other simple functions. MAALF602
6.6 ext. Interpret graphs of familiar functions and other functions such as square root, reciprocal, logarithmic and circular functions. MAALF606

Mathematical reasoning

5.1 Make, test and modify conjectures. MARSR501
5.3 Use and interpret simple mathematical models and make judgments about the accuracy and suitability of the results obtained by using mathematical models. MARSR503
6.1 Formulate and test conjectures and generalisations. MARSR601
6.4 ext. Formulate and test generalisations. MARSR604

Strategies for investigation

5.1 Generate mathematical questions for inquiry from presented data, familiar contexts and the experience gained from inquiries in relation to previous tasks and problems. MARSS501
5.3 Apply a range of strategies for inquiry to complete tasks and solve problems. MARSS503
5.4 Communicate own responses to tasks and problems appropriate for this level to others. MARSS504

Key competencies

1 Collecting, analysing and organising information
2 Communicating ideas and information
3 Planning and organising activities
4 Working with others and in teams
5 Using mathematical ideas and techniques
6 Solving problems
7 Using technology

Activity 3: Modelling with graphs of quadratic functions

This activity provides students with the opportunity to apply the knowledge they have developed in the first two activities in a modelling context. An understanding of the vertical symmetry of parabolas and the effects of a, h and k on the graph of $y = a(x - h)^2 + k$ is applied.

Assessing Activity 3

A variety of assessment strategies are available for this activity. Students prepare a report on their investigations for question 1 and at least one problem using either a multi-media or oral presentation with supporting material. Assessment criteria will

Tr

Teacher Resource

focus on the accuracy of calculations and graphs and correct determination of functions as required.

CSF learning outcomes

Equations and inequalities

6.1 Develop linear and quadratic equations and inequalities from information provided in a given context. MAALI601

Function

6.4 Use linear, quadratic and exponential functions to model data in a variety of contexts. MAALF604

6.8 ext. Use familiar functions and other functions such as square root, reciprocal, logarithmic and circular functions to model data in a variety of contexts. MAALF608

Mathematical reasoning

6.3 Make judgments about the accuracy and suitability of the results obtained by using a mathematical model. MARSR603

Strategies for investigation

6.1 Choose and use a range of strategies for inquiry when responding to tasks and problems. MARSS601

6.2 Communicate own responses to tasks and problems appropriate for this level to others. MARSS602

6.4 ext. Communicate to others, and generalise, own responses to tasks and problems. MARSS604

Key competencies

1 Collecting, analysing and organising information
2 Communicating ideas and information
3 Planning and organising activities
4 Working with others and in teams
5 Using mathematical ideas and techniques
6 Solving problems
7 Using technology

Teacher notes

Tr

Teacher Resource

Using the worksheets

The student worksheets provided are designed to assist teachers in guiding the learning process by providing necessary scaffolding for each activity. These are available as MS Word files on the CD-ROM version of this unit and can be selected, downloaded and modified as required. If preferred, rather than distributing the worksheets to students, the ideas contained in them may be incorporated into the presentation of the lesson.

Preparation

Activity 1 requires access to a computer laboratory for one lesson to do an Internet or intranet search. This provides an opportunity for students to find out for themselves that the mathematics they are learning is widely applied in a broad range of areas and helps to set the scene for the unit.

Activities 1 and 2 require students to have access to a graphing tool such as a graphics calculator or graphing software (see **Resources** page 19). If a class set of graphics calculators or a computer laboratory is not available then a view screen linked to a graphics calculator or a data show linked to a computer is suitable. Stand-alone computers can also be used with students assigned a particular computer to share or organised into groups to take turns. Ideally, students should have the opportunity to individually explore varying the values of a, h and k on the graph of $y = a(x - h)^2 + k$ and to observe the resulting graphs. If no technology is available at all for students to use or view, then some high quality printouts of suitable graphs could be used, although this approach is less than ideal.

Activity 3 has students out of their seats setting up and modelling some parabolic curves. Specific equipment requirements are listed for each problem on Worksheet 3.

Access to scientific calculators is assumed for all students throughout this unit, in particular when completing table of values and calculating the value of a from a given situation.

Teaching approaches

This unit of work takes a transformation approach to the completed square form of the quadratic function throughout each of the three activities. The completed square form allows students to easily observe the effects of varying the values of a, h and k on the graph of $y = a(x - h)^2 + k$ as well as provide the opportunity to model the graphs of quadratic functions without needing to solve sets of simultaneous equations. The extensive use of parabolic curves across a wide number of areas makes this a very relevant and interesting area of mathematics. However it is important that students engage in the Internet/intranet search in Worksheet 1.1 and have access to some form of graphing tool for the objectives to be fully achieved.

The activities have been designed to be completed in sequence. The concepts developed across the unit are self-reinforcing as students move through the activities. Some students may find that they require additional practice at certain stages and exercises from their textbook can be used to supplement the material in this unit.

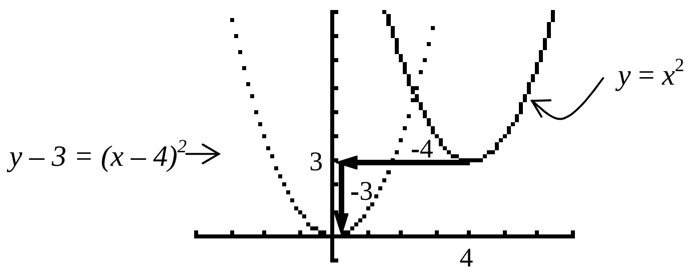
The investigation of parabolas undertaken in Activity 1 can be completed by students using appropriate search techniques on the Internet or by the use of selected sites cached on the school's intranet. For this purpose a list of suitable sites can be found in the **Resources** section (page 19).

In Worksheet 2.1 students are initially required to produce the graphs by hand. This assists in addressing the difficulties students frequently have in completing tables of values, especially for $y = {}^{-}x^2$. Completing tables of values helps students learn how to substitute values into quadratic functions, remember order of operations and deal with negative numbers. These skills will be essential for students wishing to pursue VCE Mathematical Methods Units 1–4. Plotting graphs by hand requires students to think about axes scales and enables them to appreciate the change in *y* values, which could be overlooked when using graphing tools which deal with the window settings automatically.

In using a graphics calculator to complete the remainder of Activity 1, students need to check the graphics window settings before starting. The following settings will ensure that graphs are displayed adequately: *x* minimum = ⁻7, *x* maximum = 7, *y* minimum = ⁻15 and *y* maximum = 26.

The four worksheets in Activity 2 have a skills development emphasis. Students should become aware of how each of *a*, *h* and *k* affect the graph of a quadratic function, apply this knowledge when drawing quick sketches by hand and be able to calculate the values of each, given a graph of a parabola which contains the coordinates of the vertex and one other point. This knowledge will subsequently be applied to real life contexts in Activity 3.

In this activity, teachers can address the puzzlement that students often express in why a horizontal shift to the right is associated with a negative sign, as in $y = (x - h)^2$, requiring the shift to be made in the *opposite* direction than the sign might imply, and the vertical shift made *directly* in the direction the sign implies. Students will find it useful to know that when a translation, $T_{h,k}$, is applied to the equation $y = x^2$ the resulting form of the equation is $y - k = (x - h)^2$ and that mathematical convention rewrites this equation to the form $y = (x - h)^2 + k$. It is worth noting that in the form $y - k = (x - h)^2$ one could view *the equation relating* how to get back to the origin from the image location. For example, the graph with equation $y = (x - 4)^2 + 3$, or $y - 3 = (x - 4)^2$ has its vertex at (4, 3). For this graph to be mapped back to the origin a horizontal shift of ⁻4 units and a vertical shift of ⁻3 units would be required.



Tr

Teacher Resource

Students unsure of how to proceed with the problems in Worksheet 3 will find the questions and solutions to Worksheet 2.4 helpful. A review of the minimum amount of information required to determine the rule of the quadratic function, (in these cases the vertex and one other point) will also help students take the appropriate measurements.

Suggested time allocation

The suggested time allocation for the complete unit is 10 to 13 sessions. The following table gives a time allocation for each activity. Some worksheets will need to be finished for homework if the unit of work is to be completed in two weeks.

Activity	Periods
1. Exploring parabolic curves and the basic shape of quadratic functions	2–3
2. Investigating the effect of a, h and k on the graph of $y = a(x - h)^2 + k$	5–6
3. Modelling with graphs of quadratic functions	2–3
4. Portfolio items (or test)	1

Information and communications technology (ICT)

This unit provides opportunities for students to use Information and communications technology (ICT) and to develop transferable skills. Access to computers or graphical calculators is required for all activities. For information about ICT expectations in Mathematics, refer to the Mathematics ICT charts for CSF levels 5 and 6 (see www.vcaa.vic.edu.au).

Assessing this unit

Refer to assessment details in each activity.

A portfolio of work containing the items listed below can be used to assess this unit.

- Worksheet 1.1
- a detailed summary of the key ideas developed throughout the unit, including some carefully selected examples
- fully worked solutions, including a clear graph on graph paper, of problem 1 and one other problem from Worksheet 3.

It is expected that students complete all or most of the detailed summary at home with one class period allocated for work on portfolio items. The assessment of the portfolio is criteria based, using the objectives for this unit as the criteria.

Solutions to worksheets

Worksheet 1.1

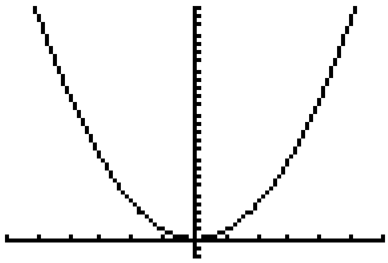
No answers.

Worksheet 1.2

1. Table of values for $y = x^2$

x	–5	–4	–3	–2	–1	–1/2	0	1/2	1	2	3	4	5
y	25	16	9	4	1	1/4	0	1/4	1	4	9	16	25

Shape of $y = x^2$



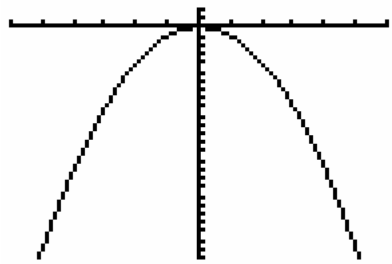
2 & 3. The parabola will continue to increase as the *x* values get larger and smaller.

4. A parabola is a curve with one turning point or vertex. It is worth pointing out to students that graphs with no turning point, or those with more than one turning point cannot be parabolas, although other curves with one turning point do exist, for example $y = x^4$ (students could graph this on the same set of axes as $y = x^2$).

5. The table of values for $y = {}^-x^2$

x	–5	–4	–3	–2	–1	–1/2	0	1/2	1	2	3	4	5
y	–25	–16	–9	–4	–1	–1/4	–0	–1/4	–1	–4	–9	–16	–25

Shape of $y = {}^-x^2$



6. The graph of $y = {}^-x^2$ is the exact same shape as the graph of $y = x^2$ reflected in the horizontal axis.

7. The parabola will continue to decrease as the *x* values get larger and smaller.

8. Parabolas have vertical symmetry.

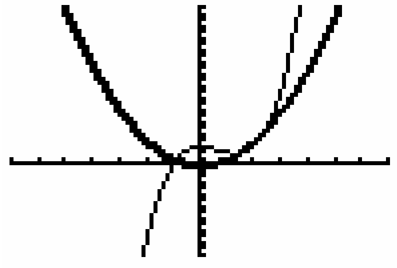
Worksheet 1.3

b), f), g) and i) are quadratic functions as the highest power associated with *x* for each is 2.

The diagram of h) is included as an example:

The thick line is the graph of $y = x^2$

The thin line is the graph of $y = 3 - 2x^2 + x^3$



Worksheet 2.1

1.

a) Table of values for $y = 2x^2$

x	–5	–4	–3	–2	–1	–1/2	0	1/2	1	2	3	4	5
y	50	32	18	8	2	1/2	0	1/2	2	8	18	32	50

b) Table of values for $y = \frac{1}{2}x^2$

x	–5	–4	–3	–2	–1	–1/2	0	1/2	1	2	3	4	5
y	12.5	8	4.5	2	1/2	1/8	0	1/8	1/2	2	4.5	8	12.5

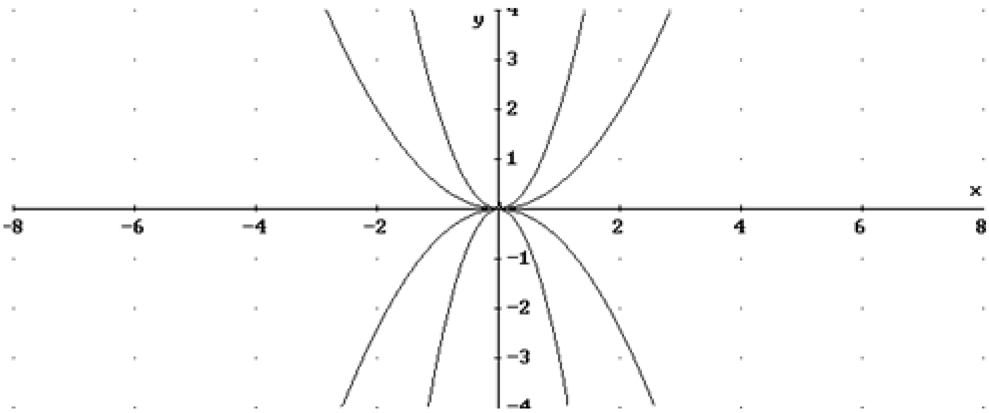
c) Table of values for $y = {}^{-}3x^2$

x	–5	–4	–3	–2	–1	–1/2	0	1/2	1	2	3	4	5
y	–75	–48	–27	–12	–3	–3/4	0	–3/4	–3	–12	–27	–48	–75

d) Table of values for $y = {}^{-}0.6x^2$

x	–5	–4	–3	–2	–1	–1/2	0	1/2	1	2	3	4	5
y	–15	–9.6	–5.4	–2.4	–0.6	–0.15	0	–0.15	–0.6	–2.4	–5.4	–9.6	–15

(b) (a)



(d) (c)

2.

The curve becomes wider, 'opening up' more with respect to the vertical axis.

The curve becomes narrower, 'tighter' with respect to the vertical axis.

The curve is reflected about the *x* axis, vertex on top, and the curve is wider and open.

The curve is reflected about the *x* axis, vertex on top, and the curve is narrower with respect to the vertical axis.

3.

When $a = 0$ the graph of $y = ax^2$ would not be a parabola but a straight line of equation $y = 0$.

4.

b) $a = 0.05$ was used to draw the graph on the worksheet. Students will obtain a range of values for *a*, values between 0.1 and 0.02 are acceptable. The equation of the satellite dish is $y = 0.05x^2$, although students' equations will vary depending on their values of *a*.

c) $a = {}^{-}2$ was used to draw the graph on the worksheet. Students will obtain a range of values for *a*, values between ${}^{-}5$ and ${}^{-}1$ are acceptable. The equation of the bridge arch is $y = {}^{-}2x^2$, although students' equations will vary depending on their values of *a*.

5 & 6.

Students who have difficulties completing these questions either have not fully understood the effect of varying the value of *a* on the graph of $y = ax^2$, or have difficulties ordering the given values of *a* from smallest to largest.

5.	6.
a) $y = 0.06x^2$	a) $y = {}^{-}0.02x^2$
b) $y = 0.1x^2$	b) $y = {}^{-}0.25x^2$
c) $y = {}^{1}/_{3}x^2$	c) $y = {}^{-}0.4x^2$
d) $y = {}^{1}/_{2}x^2$	d) $y = {}^{-}1x^2$

e) $y = 2x^2$

f) $y = 10x^2$

e) $y = {}^{-}1.2x^2$

f) $y = {}^{-}7x^2$

Worksheet 2.2

1.

a) $a = 20$, $y = 20x^2$

b) $a = {}^{-}0.5$, $y = {}^{-}0.5x^2$

c) $a = 0.2$, $y = 0.2x^2$

d) $a = {}^{-}3$, $y = {}^{-}3x^2$

e) $a = 0.25$, $y = 0.25x^2$

Worksheet 2.3

1. This question has been included to have students think about the effect h and k might have on the graph of $y = a(x - h)^2 + k$. It is not intended that they provide a detailed mathematical answer but rather comment on the change in location of the vertex from the origin.

2. Students should draw all five graphs on one sheet of graph paper. They may need to have their attention drawn to the range of values across the questions when drawing up the axes.

a) Table of values for $y = x^2 + 3$

x	–5	–4	–3	–2	–1	–1/2	0	1/2	1	2	3	4	5
y	28	19	12	7	4	3.25	0	3.25	4	7	12	19	28

b) Table of values for $y = (x - 4)^2$

x	–2	–1	0	1	2	3	4	5	6	7	8	9	10
y	36	25	16	9	4	1	0	1	4	9	16	25	36

c) Table of values for $y = (x + 2)^2 + 5$

x	–7	–6	–5	–4	–3	–2	–1	0	1	2	3	4	5
y	30	21	14	9	6	5	6	9	14	21	30	41	54

d) Table of values for $y = (x + 1)^2 - 4$

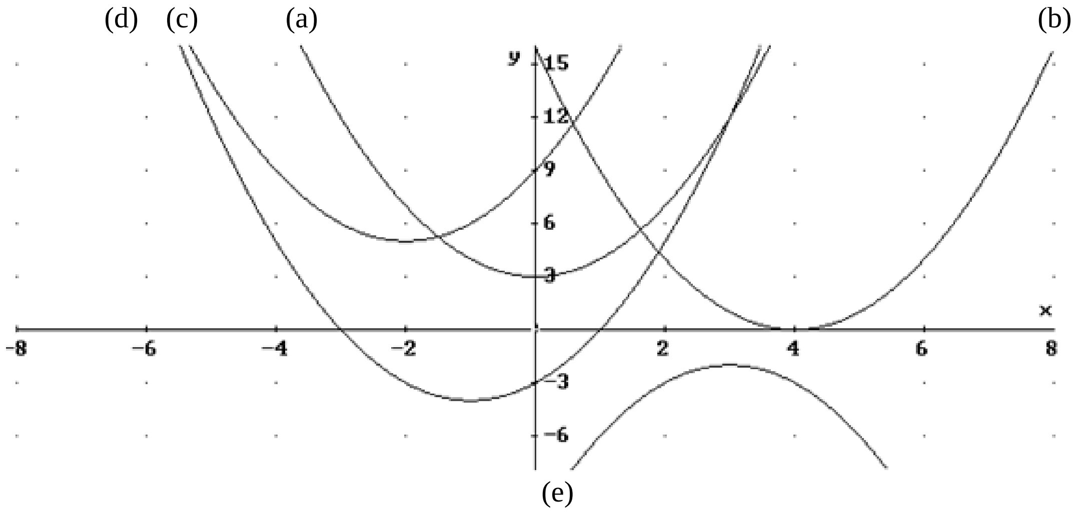
x	–6	–5	–4	–3	–2	–1	0	1	2	3	4	5
y	21	12	5	0	–3	–4	–3	0	5	12	21	32

e) Table of values for $y = {}^{-}(x - 3)^2 - 2$

x	–5	–4	–3	–2	–1	0	1	2	3	4	5	6	7
y	–66	–51	–38	–27	–18	–11	–6	–3	–2	–3	–6	–11	–18

3.

Below are the graphs of the quadratic functions from question 2.



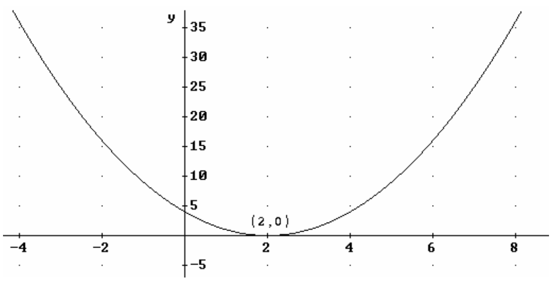
a) The value of k corresponds to the vertical shift of the graph, $k = 8$ shifts the graph 8 units up, $k = {}^{-}5$ shifts the graph 5 units down.

b) The value of h corresponds to the horizontal shift of the graph. The shift looks as though it is in the *opposite* direction to what appears in the equation. For example, $h = 4$ is a horizontal shift to the right and is embedded in the equation as $y = (x - 4)^2$. Likewise, $h = {}^{-}3$ is a horizontal shift to the left and is embedded in the equation as $y = (x + 3)^2$.

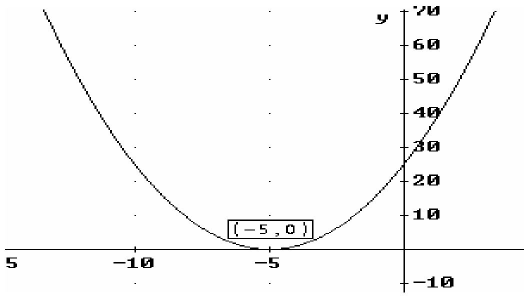
c) The coordinates of the vertex are (h, k).

4.

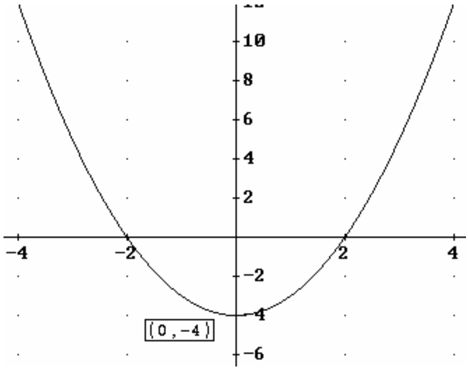
a)



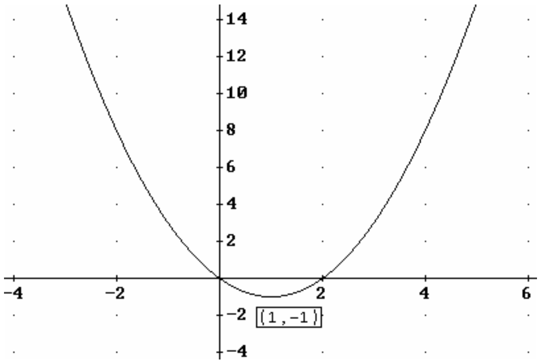
b)



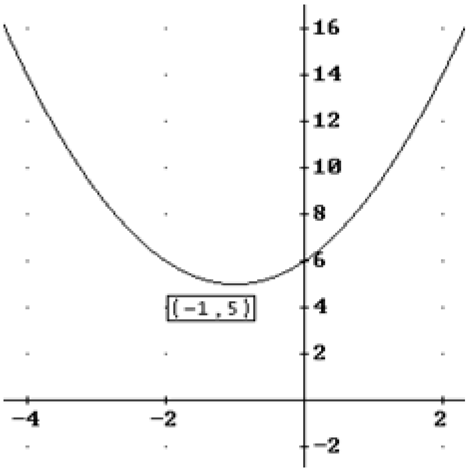
c)



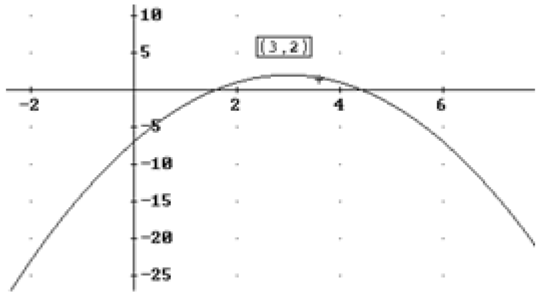
d)



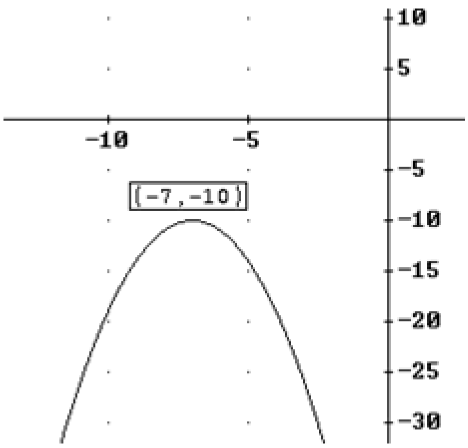
e)



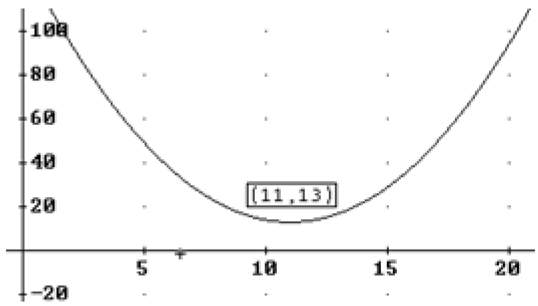
f)



g)



h)



6.

a) $a = 0.2$ $h = 0$ $k = {}^{-}3$ vertex at bottom – minimum

b) $a = {}^{-}10$ $h = 4$ $k = 0$ vertex on top – maximum

c) $a = 3$	$h = {}^{-}5$	$k = 7$	vertex at bottom – minimum
d) $a = {}^{-}0.4$	$h = 3$	$k = {}^{-}6$	vertex on top – maximum

7.

a) vertex $(0, {}^{-}6)$	$a = 1$	$y = x^2 - 6$
b) vertex $(0, {}^{-}6)$	$a = {}^{-}1$	$y = {}^{-}x^2 - 6$
c) vertex $({}^{-}4, 0)$	$a = 1$	$y = (x + 4)^2$
d) vertex $({}^{-}4, 0)$	$a = {}^{-}1$	$y = {}^{-}(x + 4)^2$
e) vertex $(3, 8)$	$a = 1$	$y = (x - 3)^2 + 8$
f) vertex $({}^{-}5, {}^{-}4)$	$a = {}^{-}1$	$y = {}^{-}(x + 5)^2 - 4$

Worksheet 2.4

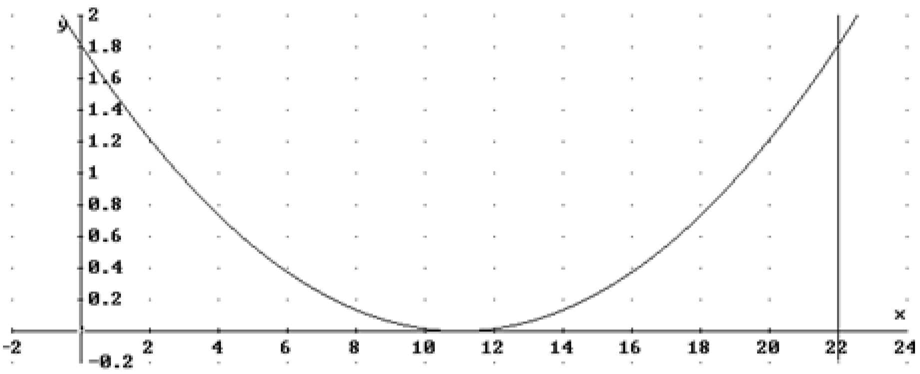
1.

a) vertex (8, 2) chosen point (0, 34) $a = 0.5$ $y = 0.5(x - 8)^2 + 2$

b) vertex $(0, {}^{-}6)$ chosen point $(1, {}^{-}8)$ $a = {}^{-}2$ $y = {}^{-}2x^2 - 6$

c) vertex (5, 3.5) chosen point (0, 0) $a = {}^{-}0.14$ $y = {}^{-}0.14(x - 5)^2 + 3.5$

d) vertex (11, 0) chosen point (0, 1.8) $a = 0.015$ $y = 0.015(x - 11)^2$. These values are based on the diagram for the curve of a suspension bridge being placed on the axes as shown below. Other diagrams are also suitable.



Worksheet 3.0

The solutions to each problem vary. However, students need to gather information about the location of the vertex and one other point in each situation. The coordinates of each will depend on where students place their curves on the set of axes. If the equation of the quadratic function matches the curve it does not matter where the origin is placed.

The location of the focal point in problem 4 is included so that students can see where the cooking plate would go, and possibly consider the practicalities of this location. Students are not to be expected to repeat these calculations in another context or on a test.

Teacher Resource

Learning outcomes

Throughout this unit, student performance can be assessed in terms of CSF Mathematics outcomes for Number, Algebra and Reasoning and data. Relevant learning outcomes for each activity are detailed in the activity notes and are also shown in the **Unit summary**.

Key competencies

Student performance can also be assessed in terms of the following key competencies. Relevant key competencies for each activity are also listed (in abbreviated form) in the activity notes and are shown in the **Unit summary**.

Collecting, analysing and organising information

The capacity to locate information, sift and sort information in order to select what is required and present it in a useful way, and evaluate both the information itself and the sources and methods used to obtain it.

Communicating ideas and information

The capacity to communicate effectively with others using a range of spoken, written, graphic and other non-verbal means of expression.

Planning and organising activities

The capacity to plan and organise one's own work activities, including making good use of time and resources, sorting out priorities and monitoring one's own performance.

Working with others and in teams

The capacity to interact effectively with other people both on a one-to-one basis and in groups, including understanding and responding to the needs of a client and working effectively as a member of a team to achieve a shared goal.

Using mathematical ideas and techniques

The capacity to use mathematical ideas, such as number and space, and techniques, such as estimation and approximation, for practical purposes.

Solving problems

The capacity to apply problem-solving strategies in purposeful ways, both in situations where the problem and the desired solution are clearly evident and in situations requiring critical thinking and a creative approach to achieve an outcome.

Using technology

The capacity to apply technology, combining the physical and sensory skills needed to operate equipment with the understanding of scientific and technological principles needed to explore and adapt systems.

Tr

Teacher Resource

Unit summary

Graphical quadratics	CSF learning outcomes	Key competencies	ICT skills
Activity 1 **Exploring parabolic curves**	• MANUM(501, 503) • MANUC503 • MAALE(501, 603) • MAALF(501, 503, 601, 602) • MARSR(501, 503) • MARSS501	• Collecting, analysing and organising information • Communicating ideas and information • Planning and organising activities • Working with others and in teams • Using mathematical ideas and techniques • Solving problems • Using technology	**Graphics calculator and CAS, Level 6** • Represents and manipulates algebraic expressions.
Activity 2 **Investigating the graph of** $y = a(x - h)^2 + k$	• MANUM501 • MANUC(503, 604) • MAALE(501, 601, 603) • MAALF(601, 602, 606) • MARSR(501, 503, 601, 604) • MARSS(501, 503, 504)	• Collecting, analysing and organising information • Communicating ideas and information • Planning and organising activities • Working with others and in teams • Using mathematical ideas and techniques • Solving problems • Using technology	**Graphics calculator and CAS, Level 6** • Represents and manipulates algebraic expressions.
Activity 3 **Modelling with graphs of quadratic functions**	• MAALI601 • MAALF(604, 608) • MARSR603 • MARSS(601, 602, 604)	• Collecting, analysing and organising information • Communicating ideas and information • Planning and organising activities • Working with others and in teams • Using mathematical ideas and techniques • Solving problems • Using technology	**Graphics calculator and CAS, Level 6 ext.** • Investigates complex and functional relationships.

Tr

Teacher Resource

Resources

Websites

At the time of publication the URLs (website addresses) cited were checked for accuracy and appropriateness of content. However, due to the transient nature of material placed on the Internet, their continuing accuracy cannot be verified. Teachers are strongly advised to prepare their own indexes of sites that are suitable and applicable to this unit of work, and to check these addresses prior to allowing student access.

Victorian Curriculum and Assessment Authority
www.vcaa.vic.edu.au

1. Websites useful for the investigation of parabolas and parabolic arches (Activity 1)

Structures in Kansai New Collection
www.kippo.or.jp/culture/gendai/evolving/bri_e.htm

Scroll down the screen until the heading 'Parabolic Arch Bridge' appears. A photo of the Kushimoto Bridge is included as well as a summary of different types of bridges. Students should notice from this that some arch bridges use semi circular rather than parabolic arches, a point which could potentially be explored further in history classes.

Young Creek (Shepperd's Dell) Bridge
www.odot.state.or.us/eshtm/shep.htm

The site gives details of the bridge. There is a photo, though it is not front-on making it harder to see the arches.

Bayonne Bridge
www.nycroads.com/crossings/bayonne/

Reference to the parabolic arches is under the 'Design and Construction' heading.

Arches, bridges
www.media.uwe.ac.uk

This site gives an overview of conic sections, the different curves that are obtained and the benefits of each one. Parabolic arches are discussed part way through.

Airship Hangars
www.uoregon.edu

This site provides a description of a parabolic airship hanger but unfortunately no image has been included.

The "Butterfly Pavillion" Exhibition Tent
www.tdrinc.com/butter.html

Impressive images of the building. Reference to the parabolic arch is made under the heading: 'General background of project'.

Wilkinson Eyre Architects
www.wilkinsoneyre.com/bridges/210%20Hulme%20Arch.htm

This site describes the Hulme Arch in Manchester and has a photograph at the end of the text section.

Tr

Teacher Resource

2. Websites for search on 'quadratics and parabolas'

Introductory Quadratics Functions Unit
www.mste.uiuc.edu/courses/ci303fa01/students/elord/quadratics.html

Overview of the key headings associated with quadratic functions.

Parabolas
www.csun.edu/~math095/schedule/notes/mod11/Parabolas.html

Highlights the intercepts for graphing parabolas as well as associated algebra.

Graphing Quadratic Equations Lesson
www.purplemath.com/modules/grphquad.htm

This site takes a slightly different approach to others. It shows how to graph the curve smoothly, why you need to consider negative values in the table of values and includes an animation for the graph $y = ax^2$, for $a = {}^{-}10$ to 10 in steps of 0.1.

Quadratic etc. equations
www-gap.dcs.st-and.ac.uk/~history/HistTopics/Quadratic_etc_equations.html

This is a more general site providing a historical perspective.

Quadratic Equation solver
www.edteach.com/algebra/quad_explorer/quadratic.html

Solves quadratic equations using a program.

3. Some other interesting sites involving parabolic curves. These were obtained by combining the word 'parabolic' with one of: cathedral, solar cooker, solar tower, skate ramp, St. Denis and comet.

CCI Insurances – Experiences
www.ccinsurances.com.au/cathedrals/cath_5.html

Has a photograph of St Mary's Cathedral in Darwin, showing the huge parabolic arch.

Occurrence of the conics
www.camosun.bc.ca/~jbritton/jbconics.htm

Shows how the different curves can be obtained from slicing the cone and then highlights numerous applications for each curve. It includes diagrams and photographs.

Solar Thermal-Electrical Energy Systems
www.acre.murdoch.edu.au/refiles/hightemp/text.html

An explanation is given of how a solar tower works as well as an image of a solar tower.

Solar Energy
www.eia.doe.gov/kids/renewable/solar.html

An easy to follow site. Contains an explanation of how a parabolic trough, solar dish, solar power tower and solar cells work.

Solar cooking plans
www.solarcooking.org/plans.htm

Shows a wide variety of solar cookers, including some based on a parabolic design.

Tr

Teacher Resource

The Solar Barbecue – Australia
www.users.bigpond.com/solarbbq/

Shows images of some solar barbecues which are available for sale.

Parabolic and nearly parabolic comets
www.makinojp.com/bekkoame/parabola.htm

Offers a list of comets which have parabolic, or nearly parabolic, paths.

www.exploremath.com

This site offers a broad range of multimedia activities and lesson plans online. There are a number of activities involving quadratic functions and parabolas, including one on quadratic functions in completed square form. To view the full range of activities available click on *activities* and then click on *full listing* at the end of the activity list. Schools which do not have access to graphics calculators or computer graphing software could use this activity to complete most of the technology graphing requirements of the unit. It could also be useful to give this URL to students who have Internet access at home.

Software

Graphics calculator or computer software with graphing functionality such as:
ANU Graph, Derive, Excel, Graphmatica, Mathematica or the like.

Student worksheets

1.1 Exploring parabolic curves and the basic shape of the graph of quadratic functions: Part 1 (pages 22–23)

1.2 Exploring parabolic curves and the basic shape of the graph of quadratic functions: Part 2 (pages 24–26)

1.3 Exploring parabolic curves and the basic shape of the graph of quadratic functions: Part 3 (page 27)

2.1 Investigating the effect of a, h and k on the graph of $y = a(x - h)^2 + k$: Part 1 (pages 28–32)

2.2 Investigating the effect of a, h and k on the graph of $y = a(x - h)^2 + k$: Part 2 (pages 33–35)

2.3 Investigating the effect of a, h and k on the graph of $y = a(x - h)^2 + k$: Part 3 (pages 36–40)

2.4 Investigating the effect of a, h and k on the graph of $y = a(x - h)^2 + k$: Part 4 (pages 41–43)

3.0 Modelling with graphs of quadratic functions (pages 44–48)

Student Worksheet

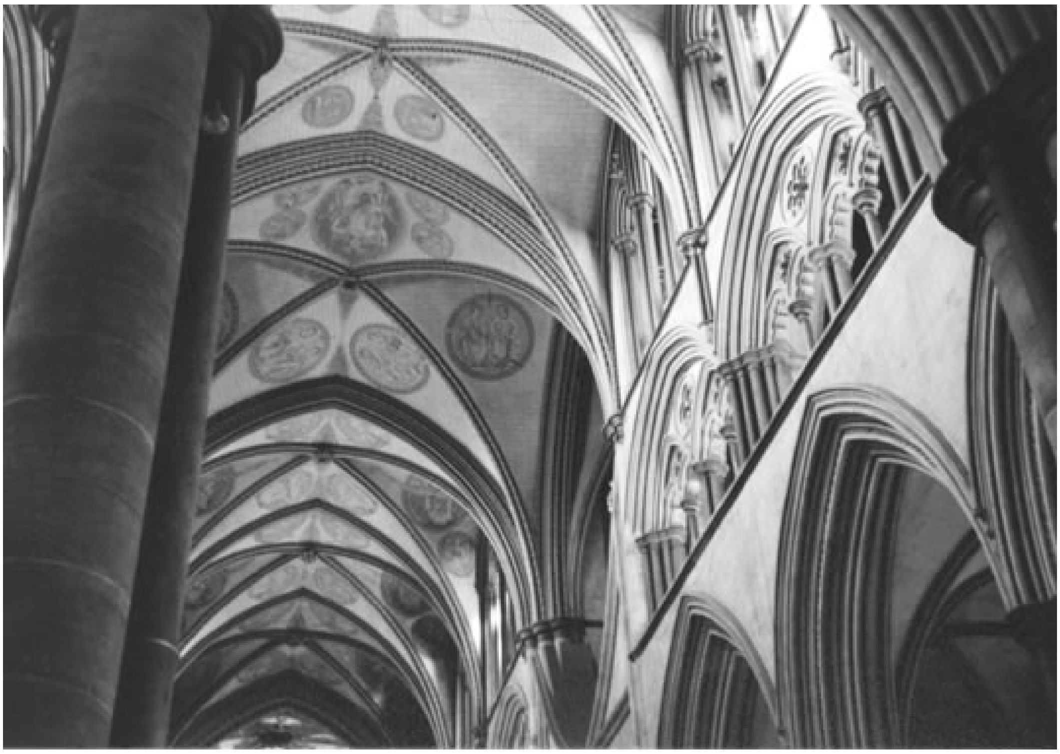
Exploring parabolic curves and the basic shape of the graph of quadratic functions: Part 1

Parabolic arches are curves formed by quadratic functions.

Perform a search on the Internet to find out more about parabolic arches and their uses.

Type in the words: parabolic arch.

Looking at the links the search has listed, what sort of areas do parabolic arches appear to be used in?



Salisbury Cathedral, England, has gothic (parabolic) arches throughout its design. Parabolic curves are good weight bearing curves.

Explore several links and answer the following questions about one of the links in detail.

1. How is the parabolic arch used?

Student Worksheet

2. If possible, draw a picture/sketch of the construction.

3. When was the construction built?

4. What are the dimensions of the parabolic arch?

For example, how long is the base of the arch from end to end and how high is the arch.

5. What materials are used in the construction?

Do another search using the words: quadratics and parabolas.

How do these links differ from those you obtained from the search on 'parabolic arch'?

Some other search possibilities are combining the word 'parabolic' with one of the following: solar cooker, cathedral, solar tower, comet.

Student Worksheet

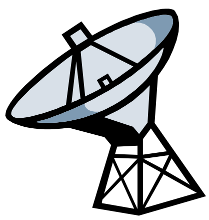
Exploring parabolic curves and the basic shape of the graph of quadratic functions: Part 2

The graph of a quadratic function has a unique shape, called a parabola, and this Worksheet will identify this shape.

The shape of a parabola is traced out by a ball being thrown up into the air.

Draw the path of a tennis ball which has been ejected from a ball machine below:

The shape of the cross section of a satellite dish is also a parabolic shape. Draw the cross section of a satellite dish below:



$y = x^2$ is the equation of the basic quadratic function.

1. Complete the table of values below for the rule $y = x^2$, which is the same as $y = x \times x$, and then graph the points with coordinates (x, y) onto a sheet of graph paper and connect the points by drawing a single smooth curve through all of them.

To calculate the value of y for $x = {}^-5$, substitute $x = {}^-5$ into the equation.

Therefore: $y = {}^-5 \times {}^-5$

$y = 25$

Student Worksheet

Table of values:

x	–5	–4	–3	–2	–1	–1/2	0	1/2	1	2	3	4	5
y	25												

2. Looking at your graph, what do you think the shape of the parabola will do as the x values get larger?

For example, consider the value of y for the following x values, $x = 7$, $x = 100$, $x = 1000$.

3. Looking at your graph, what do you think the shape of the parabola will do as the x values get smaller?

For example, consider the value of y for the following x values, $x = {}^-7$, $x = {}^-100$, $x = {}^-1000$.

4. How many turning points does the graph of a quadratic function have?

5. Complete the table of values following for the rule $y = {}^-x^2$, which is the same as $y = {}^-(x \times x)$, and then graph the points with coordinates (x, y) onto a sheet of graph paper and connect the points by drawing a single smooth curve through all of them.

Student Worksheet

To calculate the value of y for $x = {}^{-}5$, x substitute $x = {}^{-}5$ into the equation.

Therefore: $y = {}^{-}({}^{-}5 \times {}^{-}5)$

$y = {}^{-}(25)$

$y = {}^{-}25$

Table of values:

x	–5	–4	–3	–2	–1	–1/2	0	1/2	1	2	3	4	5
y	–25												

6. How does the graph of $y = {}^{-}x^2$ differ from the graph of $y = x^2$?

7. Looking at your graph of $y = {}^{-}x^2$, what do you think the shape of the parabola will do as:

   i) the x values get larger?

   ii) the x values get smaller?

8. What comments can you make about the symmetry of these parabolas?

Student Worksheet

Exploring parabolic curves and the basic shape of the graph of quadratic functions: Part 3

Using a graphing tool, such as a graphics calculator or computer software program, draw the graph of $y = x^2$ and it make it stand out by giving it a thick line or making it red.

Draw the graph of each of the following functions, while keeping the graph of $y = x^2$ on your screen, and decide if you think the graphs of the functions below are that of a quadratic function or not. The graph of each function below can be removed from your screen before drawing the next one.

a) $y = x + 3$	b) $y = x^2 + 5$	c) $y = 5$
d) $y = x^3$	e) $y = {}^{-}x - 2$	f) $y = (x - 2)^2 + 7$
g) $y = {}^{-}x^2 - 4x - 4$	h) $y = 3 - 2x^2 + x^3$	i) $y = (x - 3)(x + 6)$

Can you figure out whether a function is quadratic or not by looking at its equation?

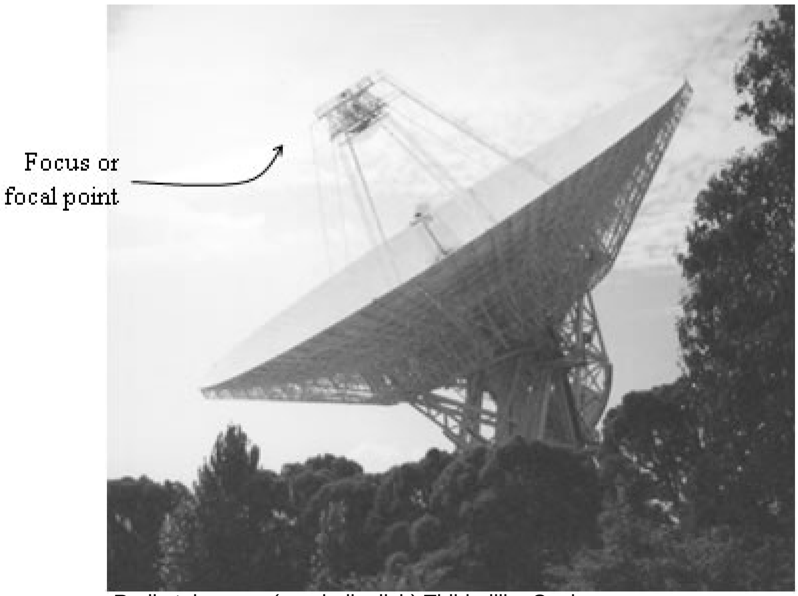
Explain.

Student Worksheet

Investigating the effect of *a*, *h* and *k* on the graph of $y = a(x - h)^2 + k$: Part 1

Activity 1 highlighted a wide range of situations in which parabolic curves are used. The basic curve, obtained by drawing the graph of $y = x^2$, does not suit all situations and often the basic curve must be dilated by varying the value of *a* to make it narrower or wider and hence meet specific design needs.

For example, in the satellite dish illustrated below, the parabolic curve is wider than the curve obtained from the graph of $y = x^2$. The dish needs to be wide to allow for a large diameter to receive signals. The signals are reflected from the dish to the focus or focal point.



Radio telescope (parabolic dish) Tidbindilla, Canberra.

The construction of arches in some bridges may require a parabolic curve narrower than the curve obtained from the graph of $y = x^2$, and with vertex at the top, as shown in the diagram below.

Student Worksheet

This Worksheet will explore the effect of varying the value of *a* in terms of dilating the graph of the basic parabola.

For the basic parabola *a* equals 1, which is why the corresponding equation is written as $y = x^2$. For other values of *a* the corresponding equation is written as $y = ax^2$, where *a* can be any real number.

1. Complete the four tables of values below for the equation $y = ax^2$ where $a = 2$, $a = {}^1/_2$, $a = {}^-3$ and $a = {}^-0.6$

Draw each graph onto the axes provided at the end of Worksheet 2.1, which has the graph of $y = x^2$ and the graph of $y = {}^-x^2$ drawn in.

a) Table of values for $y = 2x^2$

x	–5	–4	–3	–2	–1	–1/2	0	1/2	1	2	3	4	5
y													

b) Table of values for $y = {}^1/_2x^2$

x	–5	–4	–3	–2	–1	–1/2	0	1/2	1	2	3	4	5
y													

c) Table of values for $y = {}^-3x^2$

x	–5	–4	–3	–2	–1	–1/2	0	1/2	1	2	3	4	5
y													

d) Table of values for $y = {}^-0.6x^2$

x	–5	–4	–3	–2	–1	–1/2	0	1/2	1	2	3	4	5
y													

2. Comment on how the curves of the parabola for $y = ax^2$ changed with the varying values of *a*, for:

values of *a* between 0 and 1

values of *a* greater than 1

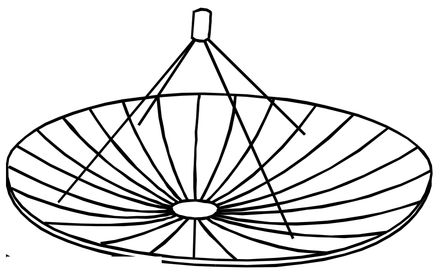
values of *a* between 0 and ${}^-1$

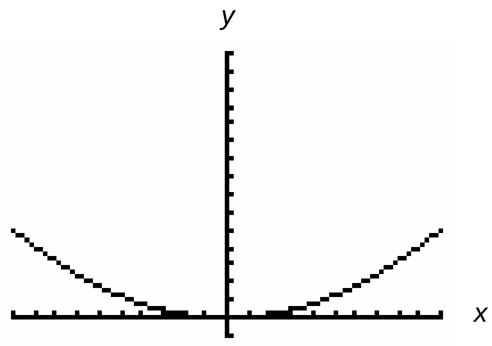
values of *a* less than ${}^-1$

3. What would the graph of $y = ax^2$ look like when $a = 0$?

4.
a) Use a graphing tool, such as a graphics calculator or computer software program to draw the graph of $y = x^2$ and $y = -x^2$. Now draw different graphs of the type $y = ax^2$, varying the value of a. Try some very small and very large positive and negative values. Draw 4–8 graphs on the same set of axes to enable comparisons to be made between them. Observe the relationship between the value of a and the curve obtained.

b) Set your graphing tool window using a minimum x value -10, maximum x value 10, minimum y value -1, maximum y value 15. Now find a value of a, for the equation $y = ax^2$, whose graph would give the shape of the satellite dish curve below. Assume the satellite dish is not yet on its stand, but is upright with the turning point (vertex) at the origin (0, 0).

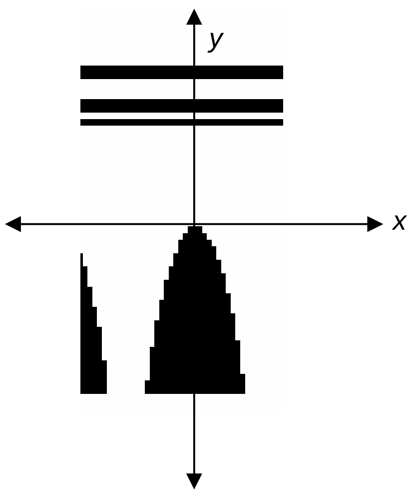




Value of a =

Equation of the satellite dish is y =

c) Set your graphing tool window using a minimum x value -10, maximum x value 10, minimum y value -25, maximum y value 1. Now find the value of a, for the equation $y = ax^2$, whose graph would give the shape of the arch on the section of bridge below. Assume the vertex of the arch is at the origin (0, 0).

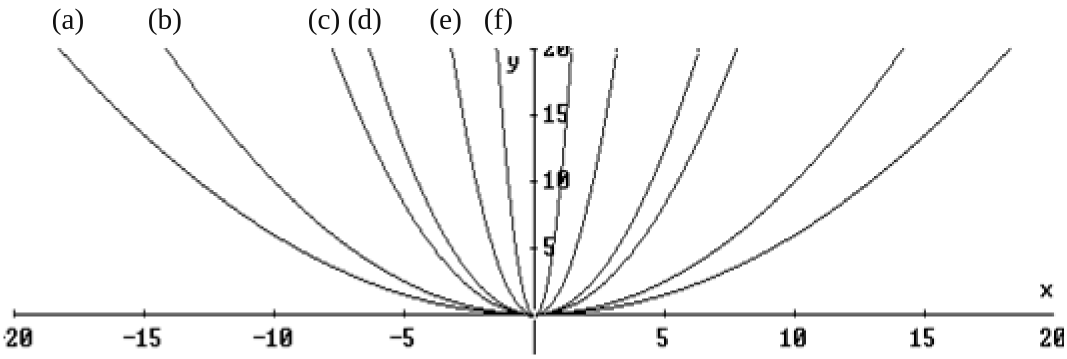


Value of a =

Equation of the arch is y =

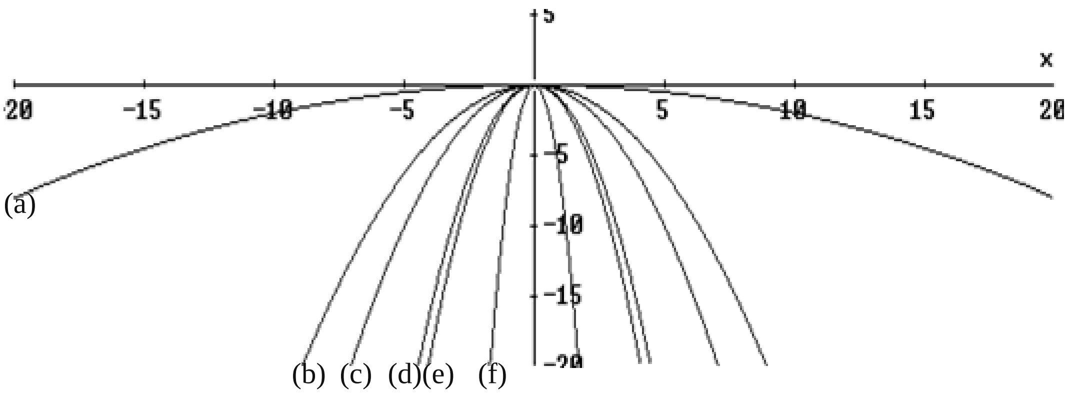
5. The equations for the parabolic curves drawn on the graph are listed below. Match each graph with the corresponding equation.

$y = 0.1x^2$ $y = {}^{1}/_{2}x^2$ $y = 10x^2$ $y = 0.06x^2$ $y = 2x^2$ $y = {}^{1}/_{3}x^2$



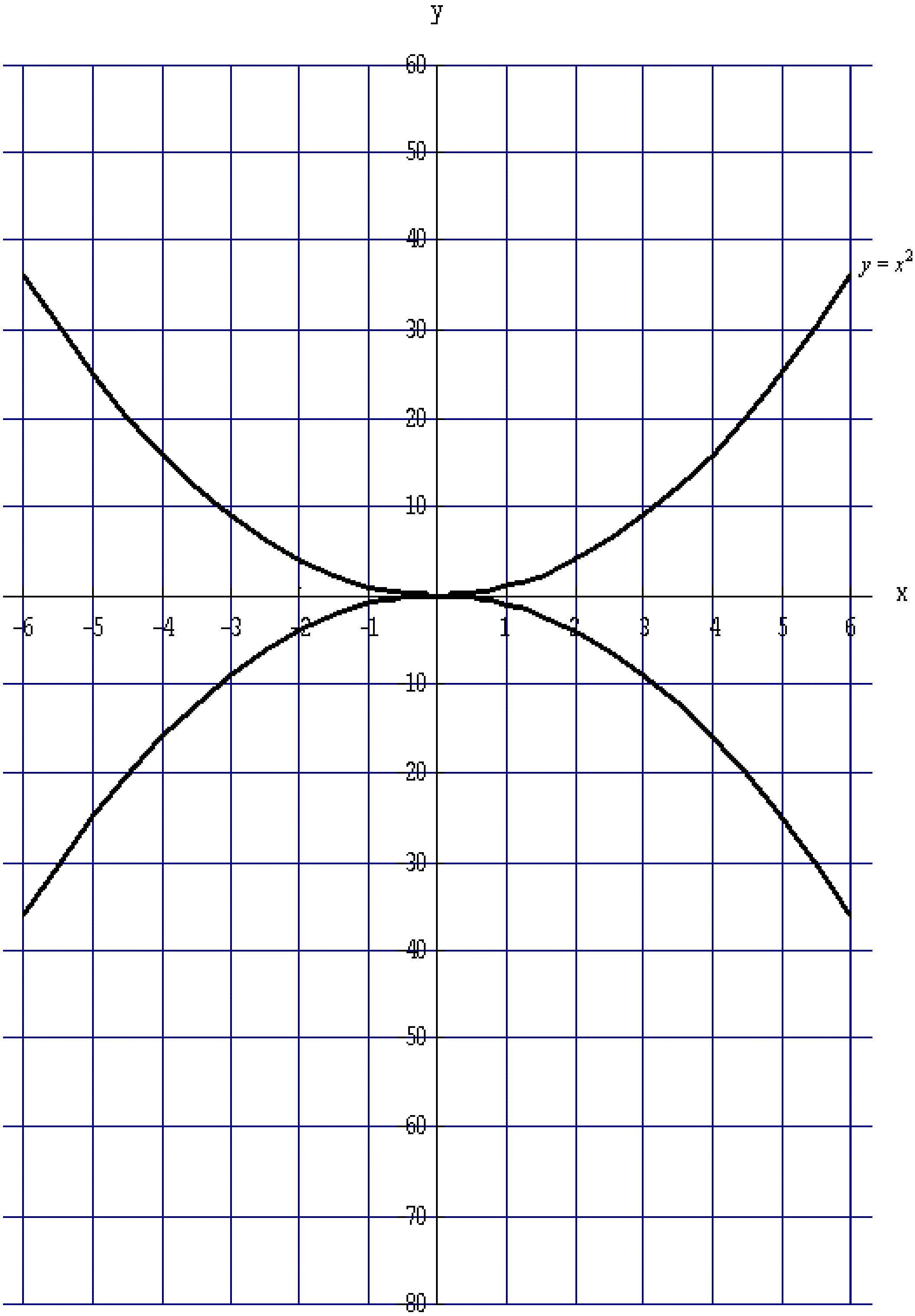
6. The equations for the parabolic curves drawn on the graph are listed below. Match each graph with the corresponding equation.

$y = {}^{-}0.4x^2$ $y = {}^{-}1.2x^2$ $y = {}^{-1}/_{4}x^2$ $y = {}^{-}7x^2$ $y = {}^{-}1x^2$ $y = {}^{-}0.02x^2$



Student Worksheet

Use this set of axes to complete question 1.

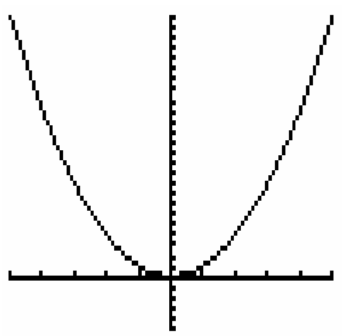


Student Worksheet

Investigating the effect of *a*, *h* and *k* on the graph of $y = a(x - h)^2 + k$: Part 2

Up until now you have been graphing parabolic curves for functions using their known equations. Is it possible to determine the equation of a curve given a graph of the curve?

Consider this graph:



We know the vertex is at the origin.

We know the equation will be of the form $y = ax^2$

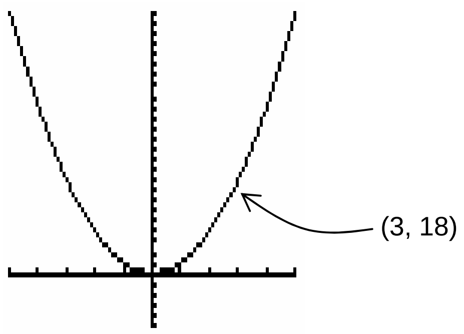
We know *a* has a positive value.

Without a scale shown on the axes it is not possible to calculate the value of *a*.

The value of *a* can be calculated if one point on the curve, other than the vertex, is known.

Example 1

The point (3, 18) belongs to the curve.



This means that now we know that for this curve when

$x = 3$, $y = 18$.

This information can be used to calculate *a* by substituting these values for *x* and *y* into the equation $y = ax^2$.

$$18 = a \times 3 \times 3$$

$$18 = 9a$$

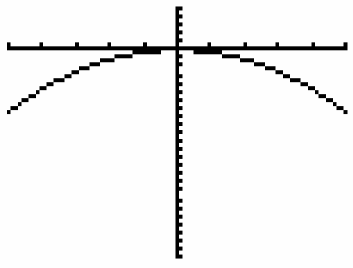
$$2 = a \text{ or } a = 2$$

Hence the equation of this curve is $y = 2x^2$.

Student Worksheet

Example 2

The point (⁻2, ⁻1.2) belongs to the curve shown:



A negative value for *a* is expected as the vertex is on top.

Substitute $x = {}^-2$ and $y = {}^-1.2$ into the equation $y = ax^2$.

$${}^-1.2 = a \times {}^-2 \times {}^-2$$

$${}^-1.2 = 4a$$

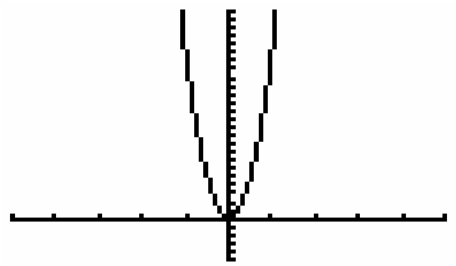
$${}^-0.3 = a \text{ or } a = {}^-0.3$$

Hence the equation of the curve is $y = {}^-0.3x^2$

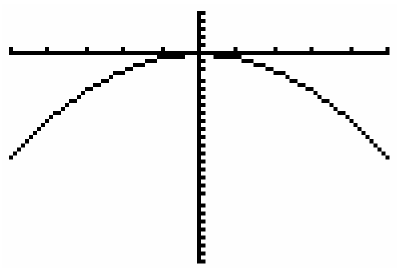
1. For each of the graphs of parabolic curves below calculate the value of *a* and then write the equation of the function corresponding to the curve.

Check your answer for *a* by looking at the diagram and estimating if the value of *a* you calculated seems reasonable.

a) The point (4, 320) belongs to this curve.

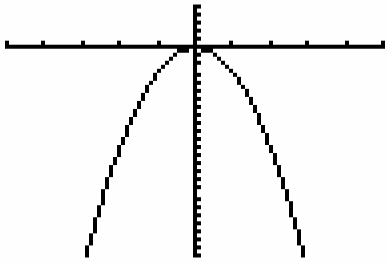


b) The point (1, ⁻0.5) belongs to this curve.

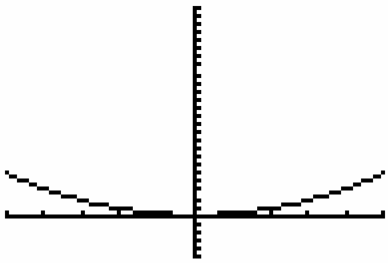


Student Worksheet

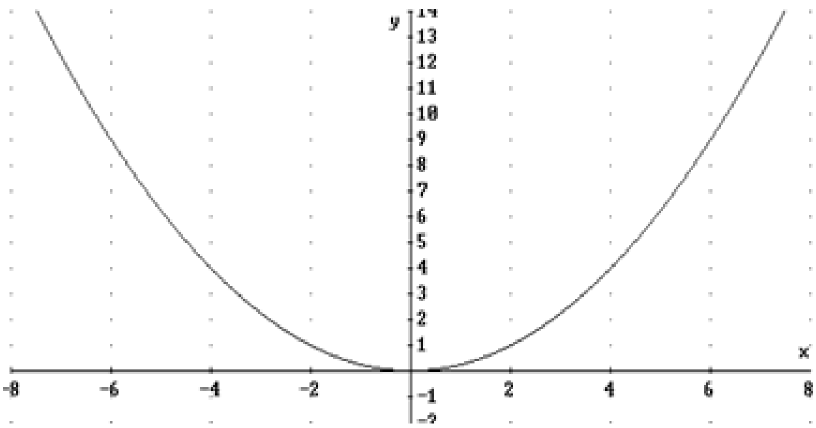
c) The point ($^{-}5$, 5) belongs to this curve.



d) The point ($^{-}3$, $^{-}27$) belongs to this curve.



e) Name a point that belongs to this curve:



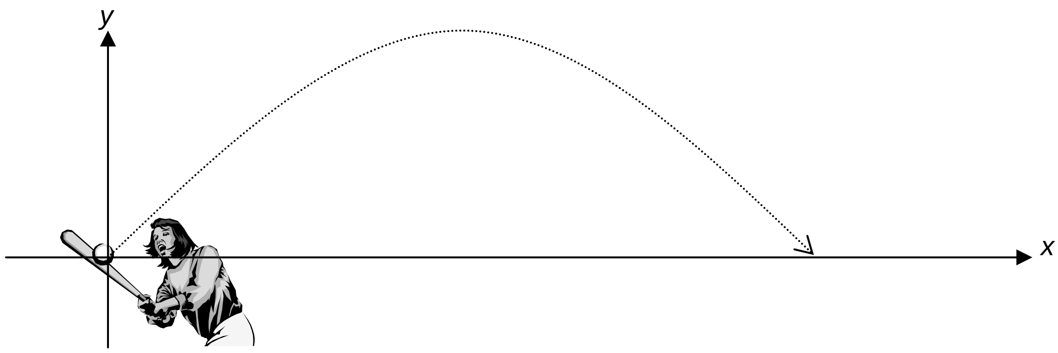
Now find the equation of the curve.

Student Worksheet

Investigating the effect of *a*, *h* and *k* on the graph of $y = a(x - h)^2 + k$: Part 3

So far you have investigated the effect of varying the values of *a* on the graph of $y = ax^2$. For each of these graphs the vertex of the parabolic curve has been at the origin (0, 0). In these instances the values of *h* and *k* in the more general equation, $y = a(x - h)^2 + k$, are zero, hence the equation simplifies to $y = ax^2$.

1. Reflect on images of parabolic curves from your Internet search and consider what effect you think varying the values of *h* and *k*, in the equation $y = a(x - h)^2 + k$, might have on the corresponding graph compared to the graph of $y = x^2$. For example, the following projectile curve is a parabola, whose rule can be described by an equation of the form $y = a(x - h)^2 + k$.



2. Complete the five tables of values below for the equation $y = a(x - h)^2 + k$, where:

a) $a = 1, h = 0, k = 3$ $y = 1(x - 0)^2 + 3$ becomes $y = x^2 + 3$

x	–5	–4	–3	–2	–1	–1/2	0	1/2	1	2	3	4	5
y	28												

b) $a = 1, h = 4, k = 0$ $y = 1(x - 4)^2 + 0$ becomes $y = (x - 4)^2$

x	–2	–1	0	1	2	3	4	5	6	7	8	9	10
y	36												

c) $a = 1, h = {}^-2, k = 5$ $y = 1(x - {}^-2)^2 + 5$ becomes $y = (x + 2)^2 + 5$

x	–7	–6	–5	–4	–3	–2	–1	0	1	2	3	4	5
y	30												

Student Worksheet

d) $a = 1$, $h = {}^{-}1$, $k = {}^{-}4$ $y = 1(x - {}^{-}1)^2 - 4$ becomes $y = (x + 1)^2 - 4$

x	–6	–5	–4	–3	–2	–1	0	1	2	3	4	5
y	21											

e) $a = {}^{-}1$, $h = 3$, $k = {}^{-}2$ $y = {}^{-}1(x - 3)^2 - 2$ becomes $y = {}^{-}(x - 3)^2 - 2$

x	–5	–4	–3	–2	–1	0	1	2	3	4	5	6	7
y	–66												

3. Draw each of the five sets of coordinates from question 2 onto one sheet of graph paper using the same set of axes for all five graphs.

What effect does varying the value of *k* have on the graph of the parabolic curve?

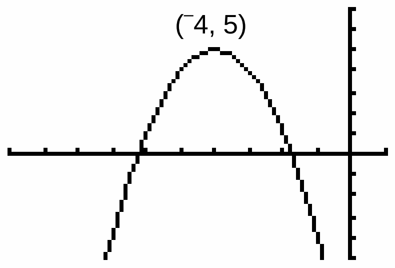
What effect does varying the value of *h* have on the graph of the parabolic curve?

The values of *h* and *k* allow you to locate which specific point on the parabolic curve?

4. For each of the following equations draw a *quick sketch* by hand of the corresponding parabolic curve, labeling the coordinates of the vertex and showing whether the vertex is on top or on the bottom.

Student Worksheet

For example:



$y = {}^{-}(x + 4)^2 + 5$

Vertex at (⁻4, 5).

a) $y = (x - 2)^2$

b) $y = (x + 5)^2$

c) $y = x^2 - 4$

d) $y = (x - 1)^2 - 1$

e) $y = (x + 1)^2 + 5$

f) $y = {}^{-}(x - 3)^2 + 2$

g) $y = {}^{-}(x + 7)^2 - 10$

h) $y = (x - 11)^2 + 13$

5. Using a graphing tool, such as a graphics calculator or computer software package:

a) Draw the corresponding graph of each of the equations in question 4 to check if your sketch is accurate.

b) Draw different graphs of the types $y = a(x - h)^2 + k$, varying the values of *a*, *h* and *k*. Draw 4–8 graphs on the same set of axes to enable comparisons to be made between them. Observe the relationship between the values of *a*, *h* and *k* and the curves obtained. See if you are able to predict what the graph will look like.

6. For each of the following equations write down the values of *a*, *h* and *k*. Then draw a *quick sketch* by hand of the corresponding parabolic curve, labelling the coordinates of the vertex, showing whether the vertex is on top or on the bottom and drawing the parabolic curve wider or narrower depending on the value of *a*.

Note: Take particular care when writing the value of *h*, why?

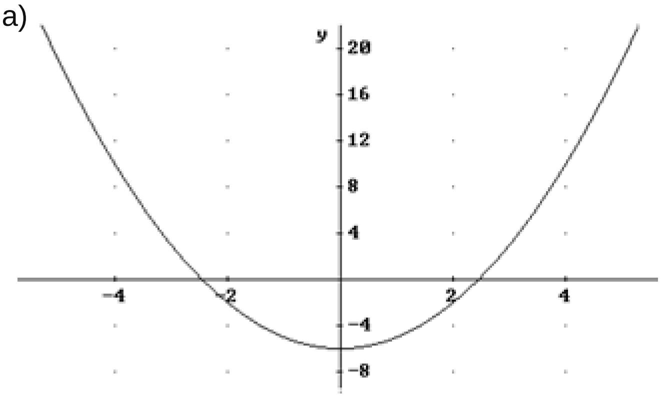
a) $y = 0.2x^2 - 3$	$a =$	$h =$	$k =$
b) $y = {}^{-}10(x - 4)^2$	$a =$	$h =$	$k =$
c) $y = 3(x + 5)^2 + 7$	$a =$	$h =$	$k =$

Student Worksheet

d) $y = {}^{-}0.4(x - 3)^2 - 6$ a = h = k =

7. Write the equation for each graph below, given the value of a for each graph is either 1 or $^{-}1$.

a)

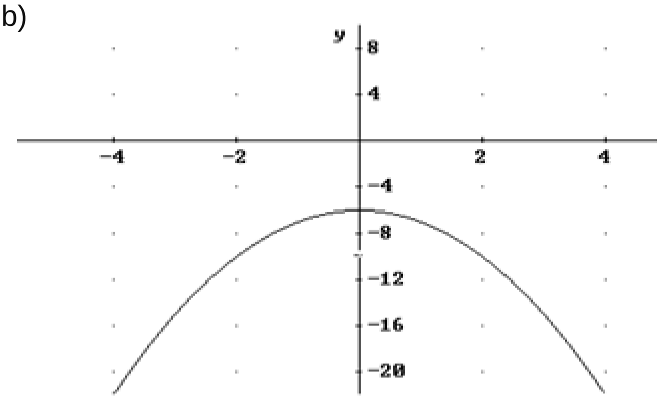


vertex: (,)

a = 1 or $^{-}1$
cross out incorrect value

y =

b)

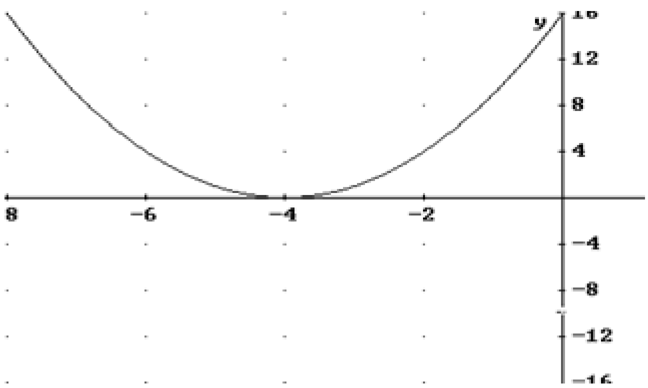


vertex: (,)

a = 1 or $^{-}1$
cross out incorrect value

y =

c)



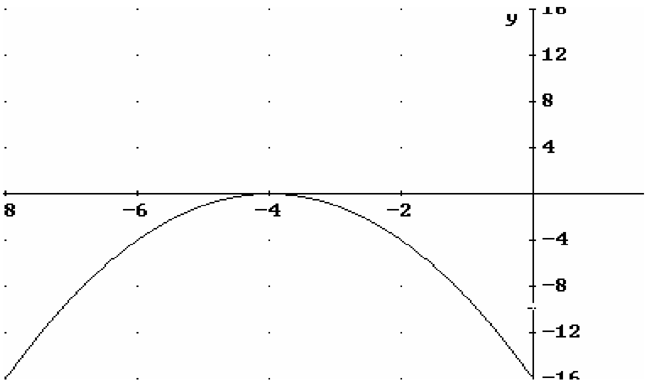
vertex: (,)

a = 1 or $^{-}1$
cross out incorrect value

y =

Student Worksheet

d)

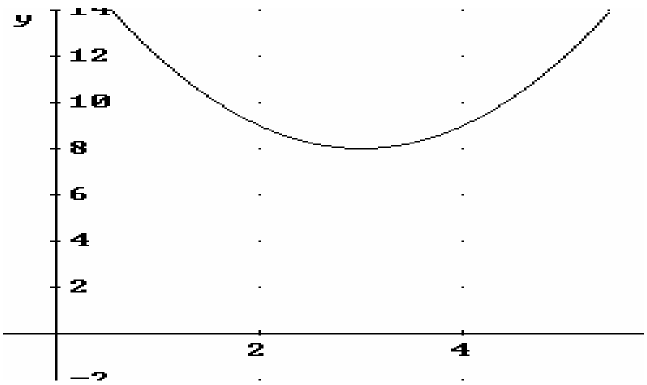


vertex: (,)

a = 1 or $^{-}1$
cross out incorrect value

y =

e)

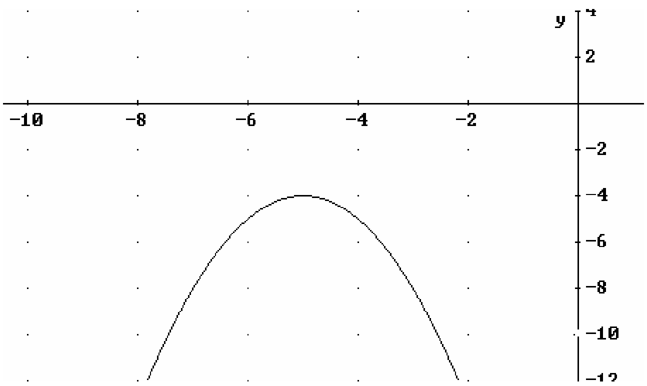


vertex: (,)

a = 1 or $^{-}1$
cross out incorrect value

y =

f)



vertex: (,)

a = 1 or $^{-}1$
cross out incorrect value

y =

Student Worksheet

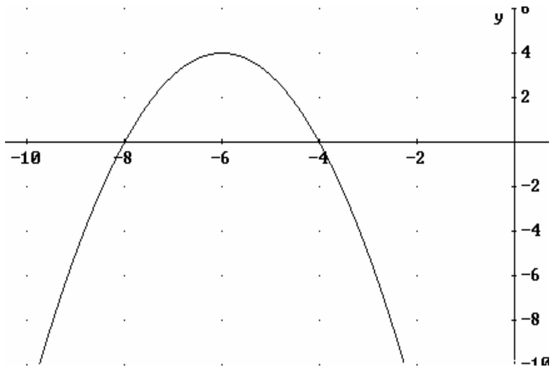
Investigating the effect of *a*, *h* and *k* on the graph of $y = a(x - h)^2 + k$: Part 4

Worksheet 2.3 looked at determining the equation of a parabolic curve when the vertex was at the origin and one other point was known, hence all equations were of the form $y = ax^2$. This task sheet will show how to determine the equation of a parabolic curve of the form $y = a(x - h)^2 + k$, when the coordinates of the vertex, not at the origin, and one other point are known.

Example

Determine the equation for this parabolic curve.

First list the coordinates of the vertex to obtain the values of *h* and *k*. Substitute these into the equation.



$y = a(x - h)^2 + k$

Vertex ($^-6$, 4)

$y = a\ (x - {}^-6)^2 + 4$

$y = a\ (x + 6)^2 + 4$

Now substitute the coordinates of an additional point into this equation to calculate the value of *a*. Any point on the curve, other than the vertex, can be chosen. Points which are whole numbers or ones which contain a zero value often make the calculations easier.

Point on the curve ($^-4$, 0).

$0 = a\ ({}^-4 + 6)^2 + 4$

$0 = a(2)^2 + 4$

$0 = 4a + 4$

${}^-4 = 4a$

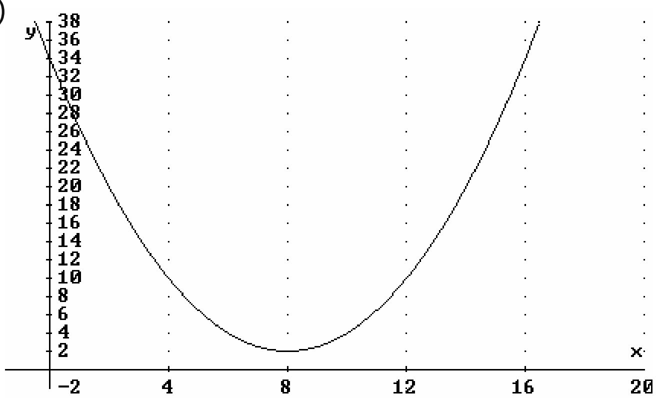
${}^-1 = a$

Therefore the equation that describes this parabolic curve is $y = {}^-1(x + 6)^2 + 4$.

Student Worksheet

1. For each of the graphs or situations involving parabolic curves below, calculate the values of *a*, *h* and *k* and then write the equation of the curve.

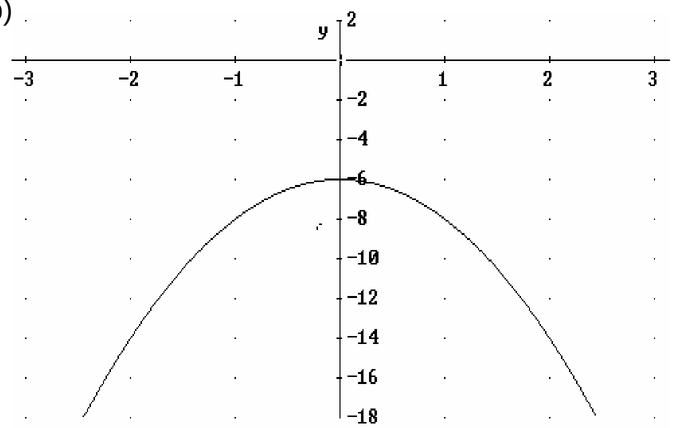
a)



Vertex (,)

Point (,)

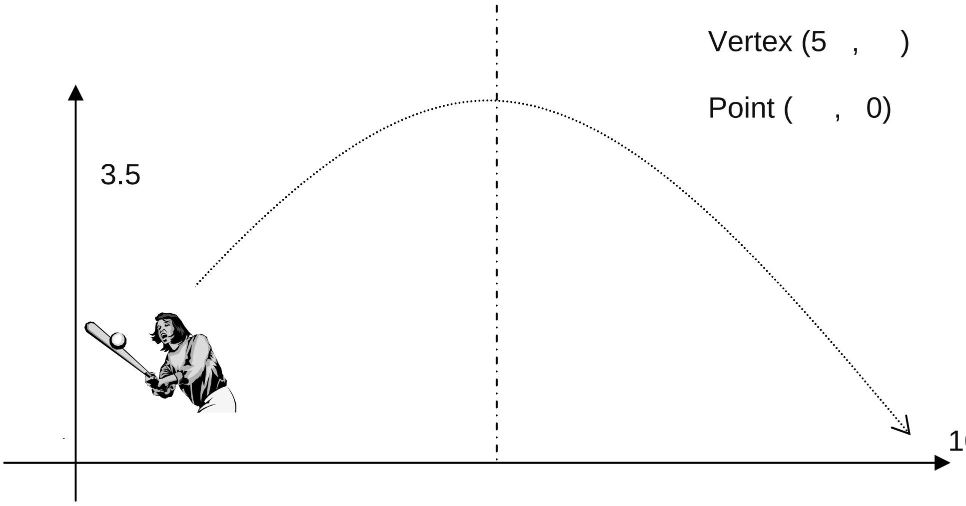
b)



Vertex (,)

Point (,)

c) A ball was hit 3.5 meters high into the air and traveled a horizontal distance of 10 metres before touching the ground.



Vertex (5 ,)

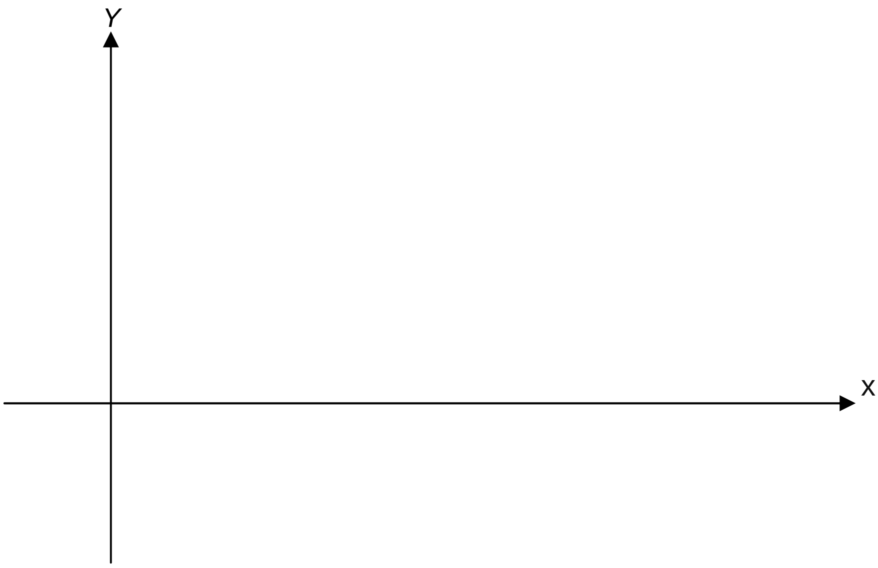
Point (, 0)

Student Worksheet

d) A suspension bridge spans a deep valley. The horizontal distance from one end of the bridge to the other is 22 metres. The deepest drop from the horizontal in the middle of the bridge is 1.8 metres.

Draw a quick sketch of the bridge on the set of axes below. Think about where you will locate the origin of the axes on your diagram and then mark on other information you know about the bridge.

Find the equation of the parabola describing the curve of the bridge.



Student Worksheet

Modelling with graphs of quadratic functions

This task sheet requires you to apply your knowledge from Activities 1 and 2 to real contexts. Four modelling contexts are presented. Each student must complete the first problem 'The curve of a piece of string' and at least one other problem from the remaining three. Solutions to two of these problems will form part of your assessment portfolio for this unit of work.

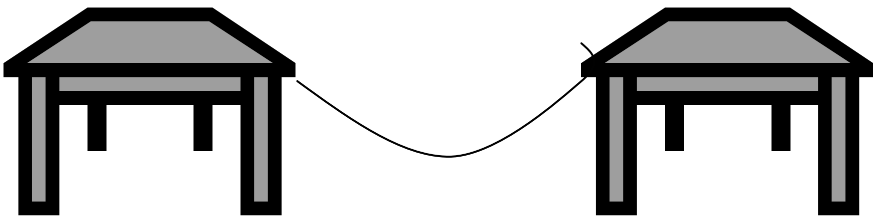
Problem 1: The curve of a suspended piece of string

Hanging ropes, chains, the wire between telegraph poles all form a curve called a catenary (Latin word for 'chain'). A catenary is a different curve to a parabola, but parabolic curves are very good approximations of catenaries. (Weight bearing hanging rope, such as that of a suspension bridge, actually forms a parabolic curve, it is the non weight bearing rope which forms a catenary).

You will model the curve of a suspended piece of string using a parabola and determine the corresponding quadratic equation.

Equipment: a piece of string 50–100cm long, sticky tape, a 1 metre ruler, two tables of the same height, graph paper.

Set up your piece of string between two tables to form a suspended curve. Use the tape to stick down the string on the table tops.



List below the sort of measurements you think you will need to make to allow you to determine the equation of the curve of the string.

Use graph paper to draw up a set of axes and hand sketch the curve you are going to model.

Make the scales on both axes a 1:1 ratio, this means that if the distance between 0 and 1 is 1cm on the *x* axis it will also be 1cm between 0 and 1 on the *y* axis. Label all important points on your graph. Give some thought to where you will place your curve in relation to the origin.

Now calculate the values of *a*, *h* and *k* and then determine the equation of the curve of the string.

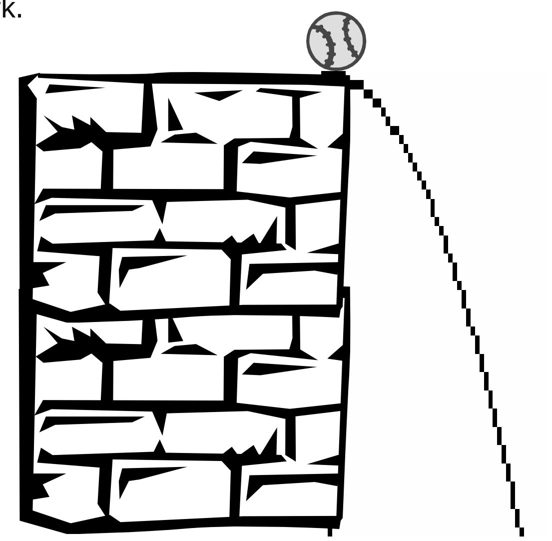
Problem 2: Path of a free falling ball

The paths of rocks falling from a cliff or a cannon ball being ejected from a cannon are parabolic.

You will model the path of a wet tennis ball being tipped off a vertical wall/fence/balcony and determine the corresponding quadratic equation.

Equipment: a wet tennis ball, a tape measure, a vertical wall, such as a fence, balcony, second storey window, for which you can measure the vertical distance from the ground.

Very gently roll the wet tennis ball of the ledge, without throwing it or really pushing it. The place where it first lands on the ground should be easily identified from the water mark.



List below the sort of measurements you think you will need to make to allow you to determine the equation of the path of the ball.

Student Worksheet

Use graph paper to draw up a set of axes and hand sketch the curve you are going to model.

Make the scales on both axes a 1:1 ratio, this means that if the distance between 0 and 1 is 1cm on the *x* axis it will also be 1cm between 0 and 1 on the *y* axis. Label all important points on your graph. Give some thought to where you will place your curve in relation to the origin.

Now calculate the values of *a*, *h* and *k* and then determine the equation of the curve of the path of the ball.

Problem 3: Footbridge

Parabolic arches have been used extensively in the construction of buildings and bridges over many years.

You are to design a wooden footbridge, whose curve is parabolic, to span a small creek 140 cm wide and determine the corresponding quadratic equation.

When designing your bridge you will need to give some thought to:

- the horizontal length of the bridge – how far from the banks of the creek will the bridge start?
- the height of the bridge – is it easy to walk over, not too steep.

Decide on the measurements of your bridge and write down your reasons for selecting them.

Student Worksheet

Use graph paper to draw up a set of axes and hand sketch the curve you are going to model.

Make the scales on both axes a 1:1 ratio, this means that if the distance between 0 and 1 is 1cm on the *x* axis it will also be 1cm between 0 and 1 on the *y* axis. Label all important points on your graph. Give some thought to where you will place your curve in relation to the origin.

Now calculate the values of *a*, *h* and *k* and then determine the equation of the curve of your bridge.

Problem 4: Solar barbecue

Parabolic curved dishes are used in satellite dishes and solar cookers as the parallel rays which come into the dish are reflected to a point in line with the center of the dish called the focus or focal point.

A solar cooker works by reflecting the suns rays from the dish to the focal point, which results in the concentration of heat at the focal point. If a cooking plate made out of conductive metal is placed at that focal point, or just below it to spread out the concentration of heat, food can be cooked on the plate when the sun is shining.

Images of some solar barbecues and cookers can be viewed at the following websites:

www.solarcooking.org/plans.htm

www.users.bigpond.com/solarbbq/

You are to design a solar cooker dish and determine the corresponding quadratic equation.

When designing your solar cooker dish you will need to give some thought to the height and width of the dish – is it still possible to stand and reach the cooking plate?

Decide on the measurements of your solar cooker dish and write down your reasons for selecting them.

Student Worksheet

Use graph paper to draw up a set of axes and hand sketch the curve you are going to model.

Make the scales on both axes a 1:1 ratio, this means that if the distance between 0 and 1 is 1cm on the *x* axis it will also be 1cm between 0 and 1 on the *y* axis. Label all important points on your graph. Give some thought to where you will place your curve in relation to the origin.

Now calculate the values of *a*, *h* and *k* and then determine the equation of the curve of your solar cooker dish.

To work out the vertical distance, *d*, of the focal point from the vertex of the parabola the following rule can be used:

$d = {}^{1}/_{4a}$

For example, for the quadratic function $y = 0.2x^2$, $a = 0.2$

Therefore: $d = \frac{1}{(4 \times 0.2)}$

$= \frac{1}{0.8}$

$= 1.25$